2 Intro to Quantitative Reasoning

QUANTITATIVE REASONING

Why Is There Math on the GRE?

To succeed in graduate school, you will need a certain baseline of mathematical skills. Academic research, even in the humanities, almost always involves quantitative information or statistical analysis of some kind.

But that is only part of it.

What's more important is that math provides a level playing field for testing something else: your ability to understand context, correctly identify a task, and infer the information required to complete that task.

No matter your field of study, you will not excel unless you can consistently understand and organize context, correctly determine your tasks or priorities, and figure out how to accomplish your tasks or get the information that you need. That's exactly what's being tested here (albeit with prime numbers and hot dog stands).

Let's take a look at some sample questions.

Sample Questions

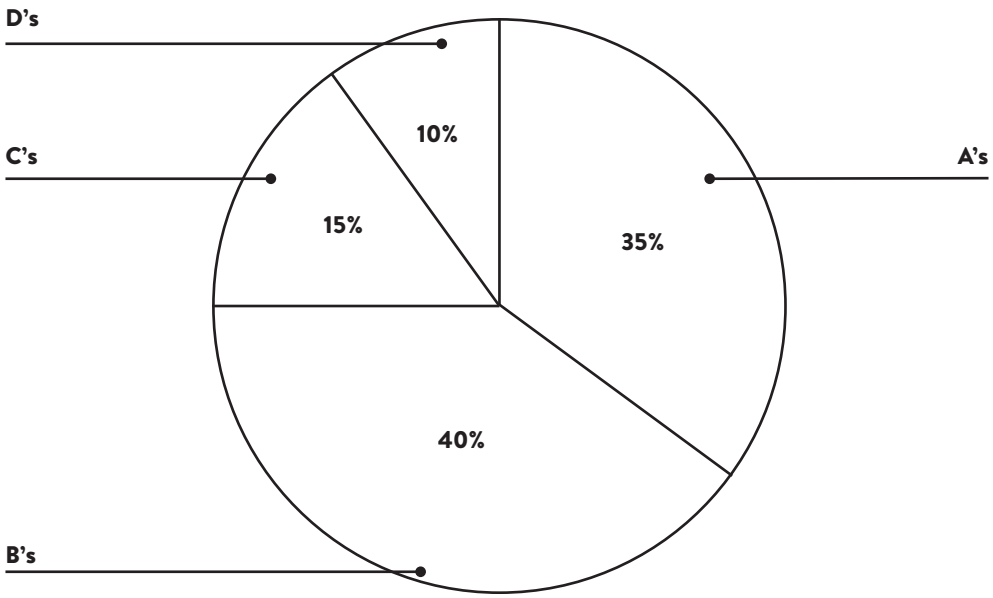
On the following pages, you'll take a look at each of the four types of questions that you'll face in the quantitative reasoning sections of the GRE: Multiple Choice (one answer), Multiple Choice (all that apply), Free Response (i.e., Numeric Entry), and Quantitative Comparison.

Please note that on the actual exam, Multiple Choice (one answer) and Quantitative Comparisons are the most common question types. Multiple Choice (all that apply) and Free Response appear significantly less often—usually just a few times per section.

Also, note that I've assigned a rough gauge of difficulty to each question for your reference, but please don't feel discouraged if you miss an easier problem or two. You might be a little rusty with some of these quantitative concepts and operations, and, well, that's what I am here for.

Multiple Choice (One Answer)

These are multiple-choice questions that ask you to select exactly one answer choice. There will always be five answers to select from, and if you ever arrive at an answer that is not among the answer choices, you will know that you've solved the question incorrectly; that is, one of the answer choices will always be correct.



A teacher has exactly 200 students in a class and distributes grades according to the above percentages. How many more students received B's than D's?

(A) 30
(B) 40
(C) 50
(D) 60
(E) 70

If two students are selected at random, the probability that neither of them received an A is approximately

(A) 22%
(B) 42%
(C) 46%
(D) 52%
(E) 65%

Multiple Choice (All that Apply)

These are multiple-choice questions that ask you to select **one or more** answers from a list of choices. Note that the question may or may not specify the exact number of choices to select, and the number of available answer choices to select from will vary from question to question. **The answers to these questions are marked with square boxes rather than ovals.**

If $x \neq 0$, which of the following is equivalent to $\frac{1}{x^2}$?

Indicate <u>all</u> such choices.

A x^{-2}
B $x^4 - x^2$
C $(x^2)^{-1}$
D $x^{-1} + x^{-1}$
E $(x^{-1})(x^{-1})$

If $(x - 3)(x - 5)$ is even, which of the following must be odd?

Indicate <u>all</u> such choices.

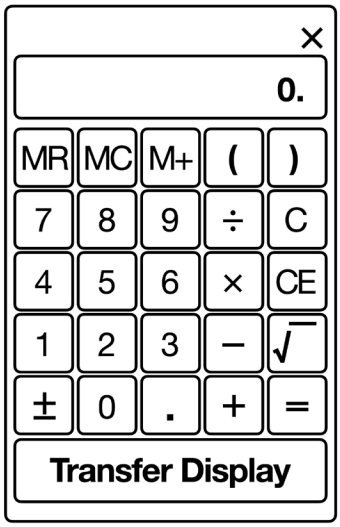
A $(x - 1)(x + 1)$
B $x(x + 3)$
C x^{x+1}
D $3x - 4(x + 1)$

On-Screen Calculator

The test makers provide an on-screen calculator that can be accessed anytime during the quantitative section by clicking the "calc" button and icon at the top of your screen.

Here are a few important notes about the operations and functions of the calculator:

- The +/– symbol converts a positive number into its negative opposite or vice versa.
- The parentheses cannot be nested; you cannot input an equation with a pair of parentheses inside another pair.
- Use "C" to clear your current equation and start fresh, or "CE" to clear only your latest input.
- You can use the memory buttons "MR," "MC," and "M+" to store, recall, or add different values while solving an equation. You don't need to use them, and in general, we recommend only doing so if you are already familiar with them.
- For Numeric Entry questions, clicking the "Transfer Display" button will transfer your result into the blank space.



Quantitative Comparison

Quantitative Comparison questions provide you with information and then ask you to compare two quantities—Quantity A and Quantity B. The answer choices for Quantitative Comparison questions are reproduced in the box to the right.

Select answer choice (A) if you can determine that Quantity A is greater than Quantity B. Select answer choice (B) if you can determine that Quantity B is greater than Quantity A. Select answer choice (C) if you can determine that the two quantities are equal. Select answer choice (D) if you cannot determine the relationship between Quantity A and Quantity B from the information that's been given to you.

(A) Quantity A is greater.

(B) Quantity B is greater.

(C) The two quantities are equal.

(D) The relationship cannot be determined from the information given.

A certain coworking space charges members a set monthly fee of $35 and an additional $17 charge for each day of use.

Quantity A	Quantity B
The total monthly charge for a member who uses the coworking space for 8 days in a particular month	$170

Ⓐ Quantity A is greater.
Ⓑ Quantity B is greater.
Ⓒ The two quantities are equal.
Ⓓ The relationship cannot be determined from the information given.

A certain tea company offers its products in two different types of packages: a cylinder with a base radius of 2 centimeters and height of 5 centimeters and a cube with side lengths of 4 centimeters. For both types of packages, the company places a label that covers all sides but not the top and not the bottom.

Quantity A	Quantity B
The area of the label on the cylindrical package	The area of the label on the cubic package

Ⓐ Quantity A is greater.
Ⓑ Quantity B is greater.
Ⓒ The two quantities are equal.
Ⓓ The relationship cannot be determined from the information given.

Free Response

Free Response, or Numeric Entry, questions ask you to simply enter your answer directly rather than selecting among given answer choices. You will have the option of inputting your answer as an integer or decimal in a single box or as a fraction in two separate boxes. Note that you are <u>not</u> required to simplify fractional responses.

By weight, liquid X makes up $\frac{1}{4}$ of solution A and $\frac{1}{10}$ of solution B. If a new solution, solution C, is created by mixing equal weights of solutions A and B, what percentage of solution C will be made up of liquid X?

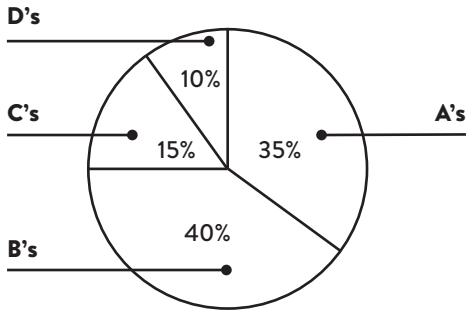
[ ] %

A certain list consists of the following 6 terms: 17, 28, 16, 25, x, $x - 6$. The mean of the list is x.

What is the value of x?

x = [ ]

Multiple Choice (One Answer) Solutions



A teacher has exactly 200 students in a class and distributes grades according to the above percentages. How many more students received B's than D's?

Ⓐ 30
Ⓑ 40
Ⓒ 50
Ⓓ 60
Ⓔ 70

Solution (Difficulty: Easy)

Notice that the chart gives you information about the percentage of students who receive each grade, but the question concerns a number of students.

According to the pie chart, 40% of students received B's, and 10% received D's. The difference between 40% and 10% is 30%, meaning that the difference between the two groups represents 30% of the total number of students.

30% of 200 = (0.3)(200) = 60

Alternatively, you could have calculated the number of students who received B's and the number of students who received D's and solved for the difference:

(0.4)(200) = 80
(0.1)(200) = 20
80 – 20 = 60

In either case, (D) is the correct answer.

If two students are selected at random, the probability that neither of them received an A is approximately

Ⓐ 22%
Ⓑ 42%
Ⓒ 46%
Ⓓ 52%
Ⓔ 65%

Solution (Difficulty: Hard)

To satisfy the given condition, you need to select two students, neither of whom received an A.

Since 35% of students received an A, for the first selection, the odds of selecting a student who did not receive an A are 65%, or 0.65, or $\frac{130}{200}$.

Once the first student is selected, assuming that the student did not receive an A, there are now $\frac{129}{199}$ students remaining who also did not receive an A.

Thus, the odds of selecting two students in a row who did not receive A's, which requires first selecting a student who did not receive an A and then, from the remaining pool, selecting another student who also did not receive an A, are:

$\left(\frac{130}{200}\right)\left(\frac{129}{199}\right)$, which is ≈ 0.42136

(B) is the correct answer.

Multiple Choice (All that Apply) Solutions

If $x \neq 0$, which of the following is equivalent to $\frac{1}{x^2}$?

Indicate all such choices.

[A] **x^{-2}**
[B] $x^4 - x^2$
[C] **$(x^2)^{-1}$**
[D] $x^{-1} + x^{-1}$
[E] **$(x^{-1})(x^{-1})$**

Solution (Difficulty: Medium)

Note that this question requires you to be fresh and familiar with exponent rules. If it's been a while, no worries. Check out the Number Properties lesson, or keep in mind that I'll cover everything you need to know when you get there.

Let's evaluate each answer choice separately.

[A] A term with a negative exponent is the reciprocal (the flipped version) of the same term with a positive exponent, so x^{-2} is equivalent to $\frac{1}{x^2}$.

[B] When you add or subtract terms with exponents, you don't add or subtract the exponents themselves. The only way to algebraically simplify this statement is to factor out x^2: $x^2(x^2 - 1)$. This is not equivalent to $\frac{1}{x^2}$.

[C] When you raise a power to a power, you multiply the exponents together. So, $(x^2)^{-1} = x^{-2}$, or $\frac{1}{x^2}$, which is the same as the original statement.

[D] $x^{-1} + x^{-1} = \frac{1}{x} + \frac{1}{x} = \frac{2}{x}$. This is not equivalent to $\frac{1}{x^2}$.

[E] When you multiply exponential terms, you add their exponents. So, in this case, $(x^{-1})(x^{-1}) = x^{-2}$, or $\frac{1}{x^2}$, and this is equivalent to the original statement.

Therefore, answer choices [A], [C], and [E] are all correct.

If $(x - 3)(x - 5)$ is even, which of the following must be odd?

Indicate all such choices.

[A] $(x - 1)(x + 1)$
[B] $x(x + 3)$
[C] **x^{x+1}**
[D] **$3x - 4(x + 1)$**

Solution (Difficulty: Hard)

If $(x - 3)(x - 5)$ is even, that means that $x - 3$ and $x - 5$ must be even, meaning x itself must be odd.

If x were even, $x - 3$ and $x - 5$ would both be odd, in which case the resulting product would also be odd. Here are some examples to illustrate.

If x were 7: $(7 - 3)(7 - 5) = (4)(2) = 8$, which is even.
If x were 6: $(6 - 3)(6 - 5) = (3)(1) = 3$, which is odd.

Thus, x must be odd, and this is the key inference that unlocks this question. Knowing that x is odd, let's evaluate the answer choices.

[A] When x is odd, $(x - 1)$ is even, and $(x + 1)$ is even. Even times even is even, so [A] does not have to be odd.

[B] Odd + 3 = even, and odd times even is even, so [B] does not have to be odd.

[C] An odd number taken to any positive integer exponent will be odd (imagine, for example, 3, 9, 27, 81, 243…), so [C] must be odd.

[D] If x is odd, $3x$ must be odd. $4(x + 1)$ must be even. Odd minus even is odd, so [D] must be odd.

[C] and [D] must be odd.

Quantitative Comparison Solutions

A certain coworking space charges members a set monthly fee of $35 and an additional $17 charge for each day of use.

Quantity A	Quantity B
The total monthly charge for a member who uses the coworking space for 8 days in a particular month	$170

Ⓐ **Quantity A is greater.**
Ⓑ Quantity B is greater.
Ⓒ The two quantities are equal.
Ⓓ The relationship cannot be determined from the information given.

Solution (Difficulty: Easy)

For Quantity A, since the person is using the coworking space for 8 days, the total charge for the month would equal:

$35 + 17(8) = 35 + 136 = 171$

Thus, Quantity A is greater than Quantity B.

A certain tea company offers its products in two different types of packages: a cylinder with a base radius of 2 centimeters and height of 5 centimeters and a cube with side lengths of 4 centimeters. For both types of packages, the company places a label that covers all sides but not the top and not the bottom.

Quantity A	Quantity B
The area of the label on the cylindrical package	The area of the label on the cubic package

Ⓐ Quantity A is greater.
Ⓑ **Quantity B is greater.**
Ⓒ The two quantities are equal.
Ⓓ The relationship cannot be determined from the information given.

Solution (Difficulty: Medium)

If the cylinder has a radius of 2 centimeters, its circumference ($2\pi r$) is 4π centimeters, and the area of its label would thus be 20π centimeters, or approximately 62.8 square centimeters.

For the cube, the height of the label would be 4 centimeters, and the length would be 16 centimeters, for an area of 64 square centimeters.

The label on the cubic package would be larger, so Quantity B is greater than Quantity A.

Free Response Solutions

By weight, liquid X makes up $\frac{1}{4}$ of solution A and $\frac{1}{10}$ of solution B. If a new solution, solution C, is created by mixing equal weights of solutions A and B, what percentage of solution C will be made up of liquid X?

17.5	%

Solution (Difficulty: Medium)

Since you are adding equal weights of each solution, the resulting mixture will be 50% solution A and 50% solution B. So, the amount of liquid X will be $\frac{1}{4}$ of 50% plus $\frac{1}{10}$ of 50%:

$\frac{1}{4}$ of 50% = 12.5%

$\frac{1}{10}$ of 50% = 5%

12.5% + 5% = 17.5%

17.5% is the correct answer.

A certain list consists of the following 6 terms: 17, 28, 16, 25, x, $x - 6$. The mean of the list is x.

What is the value of x?

$x =$ **20**

Solution (Difficulty: Medium)

If the average of all the terms is x, you can set up the following equation and solve for x:

$$\frac{17 + 28 + 16 + 25 + x + x - 6}{6} = x$$

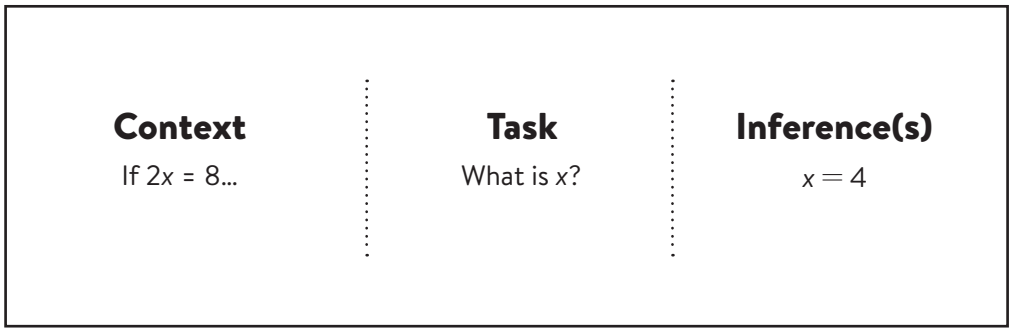
$17 + 28 + 16 + 25 + x + x - 6 = 6x$
$86 + 2x - 6 = 6x$
$80 = 4x$
$20 = x$

The correct answer is 20.

The Design of GRE Questions

You can think of all GRE quantitative questions, regardless of type, as being designed in the same way:

1. The question provides you with **context**.
2. It then presents you with a **task**.
3. In order to accomplish this task, you have to make some sort of **inference**.



Of course, certain questions are easier, and others are more challenging. Certain questions make your task obvious, and others cloak it a bit more. However, thinking about all GRE quantitative questions in this way can help clarify your habits and categorize your challenges.

How to Study GRE Quantitative Reasoning

To answer any one particular question correctly, you need to:

1. Accurately assess the context.
2. Gain a clear sense of the task being presented.
3. Be comfortable with the math principles involved.
4. Find the missing pieces of information required to complete the task.

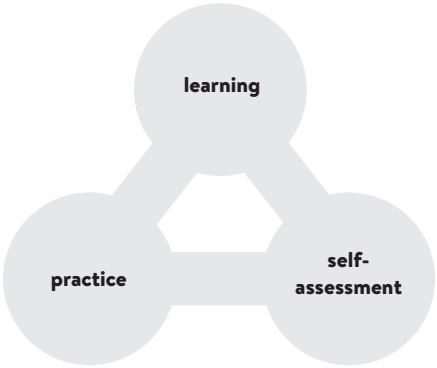
Why might you miss questions?

1. You misunderstand the context.
2. You identify the wrong task or lose sight of the task during your process.
3. You are uncomfortable or unfamiliar with the math principles involved.
4. You can't find the missing pieces of information required to complete the task.

How to Study Effectively

Developing effective study habits leads to consistently strong performance. As I've discussed, this process involves a combination of three components:

- **Learning** about the nuances of GRE questions and, most important, about quantitative topics.
- **Practice** topic-specific drills or question sets and integrated practice sections or full exams.
- **Self-Assessment**, which involves earnest review, careful consideration of strengths and weaknesses, and decision-making about the work that needs to be done to achieve mastery.



As discussed, the key to efficient and effective studying is coordinating these components—learning, practice, and assessment—so that they influence each other positively. On the flip side, be careful of disconnects between your learning, practice, and assessment. Do all you can to integrate the three.

Suggestion: Keep an Error Log

I strongly suggest keeping a running list of the questions you missed during your studies (perhaps in your GRE notebook) and reviewing this list periodically. You can take notes on each question: Did you have difficulty understanding the context, getting a sense of your task, or completing your task? What was it about the question that made it a challenge? And then, of course, once you have the proper understanding and experience, you can practice solving the question in a correct and ideal manner. Keep coming back to your error log, and keep reviewing each question that caused you trouble until you find that the questions no longer cause you any trouble.

Keeping such a log serves multiple benefits. Obviously, it can help you recognize and address isolated and unique concerns. Beyond that, it can help you notice patterns that you may not notice otherwise, and as test day approaches, it can give you a much stronger sense that you've covered all your bases and addressed all your weaknesses.

Context, Task, Inference

Let's quickly discuss how two of the previous questions fit into our framework for how GRE quantitative questions are designed.

A certain coworking space charges members a set monthly fee of $35 and an additional $17 charge for each day of use.

Quantity A	Quantity B
The total monthly charge for a member who uses the coworking space for 8 days in a particular month	$170

Ⓐ Quantity A is greater.
Ⓑ Quantity B is greater.
Ⓒ The two quantities are equal.
Ⓓ The relationship cannot be determined from the information given.

Context: You are told that the certain coworking space charges a set monthly fee of $35 and an additional $17 charge for each day of use, that Quantity A is equal to the total monthly membership charge for someone who uses the coworking space for 8 days in a particular month, and that Quantity B is equal to $170.

Task: You need to figure out whether you have enough information to know which value is greater, and, if so, which value is greater.

The Key Inference(s): Since you know Quantity B, you need to see if you can determine Quantity A, and per the information given in the question stem, you can.

$35 + 17(8) = 35 + 136 = 171$

Therefore, you can determine that Quantity A is greater and (A) is the correct answer.

If $(x - 3)(x - 5)$ is even, which of the following must be odd?

Indicate all such choices.

A $(x - 1)(x + 1)$
B $x(x + 3)$
C x^{x+1}
D $3x - 4(x + 1)$

Context: You are told that $(x - 3)(x - 5)$ is even.

Task: You need to identify which of the answer choices is odd.

The Key Inference(s): The key inference is that x is odd.

Knowing that x is odd, you can evaluate each answer choice individually, and once you do so, **you find that answer choices [C] and [D] must all be true**. *(Please see the problem solution for further discussion.)*
